MOTION CONTROL MECHANICS

The mechanical drive systems used in servo motion control applications can be divided into 4 basic categories: direct drive, gear drive, leadscrew drive, and tangential drive. This application note describes each of these drive categories and outlines the relevant formulas for calculating the various load parameters. In all instances, the formulas reflect all parameters back to the motor shaft. This means that all load parameters are transformed to the equivalent load parameters "seen" by the motor. Reflecting all parameters back to the motor shaft eases the calculations necessary for sizing the motor and controller.

The first section of this note reviews the formulas for calculating the inertia of a cylinder. These formulas are important since inertia is a key load parameter and the inertia of many mechanical components (e.g. leadscrews, gears, shafts, drive rollers, etc.) can be calculated using this formula.

CALCULATING THE INERTIA OF A CYLINDER

Inertia is the resistance of an object to be accelerated or decelerated. In motion control applications, inertia is an important parameter since it defines the torque required to accelerate and decelerate the load.

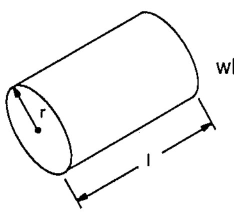
SOLID CYLINDER

The inertia of a solid cylinder can be calculated if either its weight and radius or its density, radius, and length are known.

For known weight and radius: $J_L = \frac{1}{2}\frac{W}{g}r^2 = (0.0013)Wr^2$

For known density, radius, and length:

$$J_L = \frac{1}{2}\frac{\pi lpr^4}{g} = (0.0041)lpr^4$$



where:
- J_L = inertia (lb-in-s²)
- W = weight (lb)
- r = radius (in)
- l = length (in)
- p = density of material (lb/in³)
- g = gravitational constant (386 in/s²)

HOLLOW CYLINDER

The inertia of a hollow cylinder can be calculated if its weight, inside radius, and outside radius are known or if its density, inside radius, outside radius, and length are known.

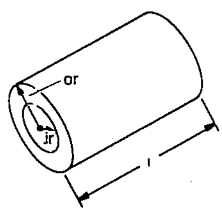
The densities of some commonly used materials are given in the table at the top in the next column.

For known weight and radii:

$$J_L = \frac{1}{2}\frac{W}{g}(or^2 + ir^2) = (0.0013)(or^2 + ir^2)W$$

For known density, radii, and length:

$$J_L = \frac{\pi}{2}\frac{lp}{g}(or^4 - ir^4) = (0.0041)(or^4 - ir^4)lp$$



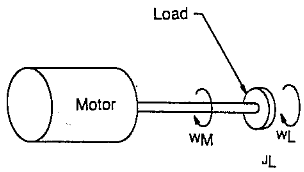
where:
- J_L = inertia (lb-in-s²)
- W = weight (lb)
- or = outside radius (in)
- ir = inside radius (in)
- l = length (in)
- p = density of material (lb/in³)
- g = gravitational constant (386 in/s²)

MATERIAL DENSITIES

Material	lb/in³
Aluminum	0.096
Brass	0.300
Bronze	0.295
Copper	0.322
Steel (cold rolled)	0.280
Plastic	0.040
Hard Wood	0.029
Soft Wood	0.018

DIRECT DRIVE LOAD

For direct drive loads, the load parameters do not have to be reflected back to the shaft since there are no mechanical linkages involved.



Speed: $w_M = w_L$

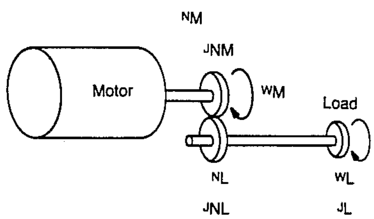
Torque: $T_L = T'$

Inertia: $J_T = J_L + J_M$

where:
- w_M = motor speed (rpm)
- w_L = load speed (rpm)
- J_T = total system inertia (lb-in-s²)
- J_L = load inertia (lb-in-s²)
- J_M = motor inertia (lb-in-s²)
- T_L = load torque at motor shaft (lb-in)
- T' = load torque (lb-in)

GEAR DRIVEN LOAD

Load parameters in a gear driven system have to be reflected back to the motor shaft by the gear ratio or the gear ratio squared. The inertia of the gears has to be included in the calculations. The gear inertias can be calculated using the formulas for the inertia of a cylinder.



Speed: $w_M = w_L(N_L/N_M)$

Torque: $T_L = T'(N_M/N_L)$

Inertia: $J_T = (N_M/N_L)^2(J_L + J_{NL}) + J_M + J_{NM}$

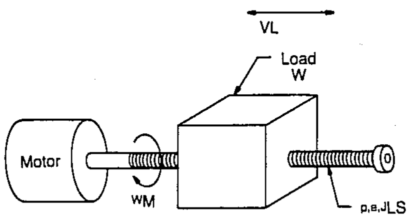
where:
- w_M = motor speed (rpm)
- w_L = load speed (rpm)
- N_M = number of motor gear teeth
- N_L = number of load gear teeth
- T_L = load torque reflected to motor shaft (lb-in)
- T' = load torque (lb-in)—not reflected
- J_T = total system inertia (lb-in-s²)
- J_L = load inertia (lb-in-s²)
- J_M = motor inertia (lb-in-s²)
- J_{NM} = motor gear inertia (lb-in-s²)
- J_{NL} = load gear inertia (lb-in-s²)

LEADSCREW DRIVEN LOAD

For this type of drive system, the load parameters have to be reflected back to the motor shaft. The inertia of the leadscrew has to be included and can be calculated using the formulas for inertia of a cylinder. For precision positioning applications, the leadscrew is sometimes preloaded to eliminate or reduce backlash. If preloading is used, the preload torque must be included since it can be a significant term. The leadscrew's efficiency must also be considered in the calculations. Efficiencies of various types of leadscrews are shown here.

TYPICAL LEADSCREW EFFICIENCIES

Type	Efficiency
Ball-nut	0.90
Acme with plastic nut	0.65
Acme with metal nut	0.40



Speed: $w_M = v_L p$

Torque: T_L⚠ $= \dfrac{1}{2\pi}\dfrac{F_L}{pe} + \dfrac{1}{2\pi}\dfrac{F_{PL}}{p} \times 0.2$

$= (0.159)F_L/pe + (0.032)F_{PL}/p$

Inertia: $J_T = \dfrac{W}{g}\left(\dfrac{1}{2\pi p}\right)^2 \dfrac{1}{e} + J_{LS} + J_M$

$= (6.56 \times 10^{-5})W/ep^2 + J_{LS} + J_M$

Friction: $F_F = uW$

$T_F = \dfrac{1}{2\pi}\dfrac{F_F}{pe} = (0.159)F_F/pe$

where:
- w_M = motor speed (rpm)
- v_L = linear load speed (in/min)
- p = lead screw pitch (revs/in)
- e = lead screw efficiency
- T_L = load torque reflected to motor shaft (lb-in)
- T_F = friction torque (lb-in)
- F_L = load force (lb)
- F_{PL} = preload force (lb)
- J_T = total system inertia (lb-in-s^2)
- J_M = motor inertia (lb-in-s^2)
- J_{LS} = lead screw inertia (lb-in-s^2)
- W = load weight (lb)
- F_F = frictional force (lb)
- u = coefficient of friction
- g = gravitational constant (386 in/s^2)

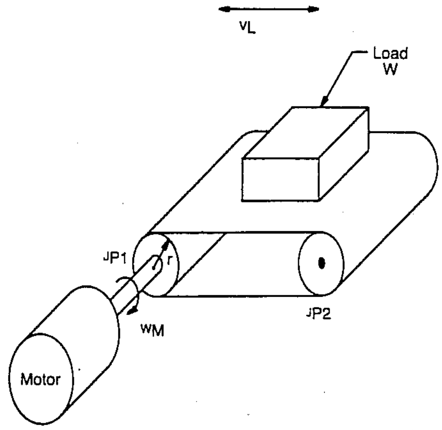
COEFFICIENTS OF FRICTION

Steel on steel	0.580
Steel on steel (lubricated)	0.150
Teflon on steel	0.040
Ball bushing	0.003

⚠ For certain applications, the frictional drag torque due to preloading should also be considered as part of the total torque requirements. Since optimum preloading is one-third of operating load, it is common practice to use 0.2 as the preload torque coefficient for the ball screw to obtain a maximum figure for preload frictional drag torque. At higher than optimum preloading, the preload frictional drag will add to the torque requirements, since it is a constant.

TANGENTIALLY DRIVEN LOAD

For this type of drive system, the load parameters have to be reflected back to the motor shaft. A tangential drive can be a rack and pinion, timing belt and pulley, or chain and sprocket. The inertia of the pulleys, sprockets, or pinion gears must be included in the calculations.



Speed: $w_M = \dfrac{1}{2\pi}\dfrac{V_L}{r} = (0.159)v_L/r$

Torque: $T_L = F_L r$

Inertia: $J_T = \dfrac{W}{g} r^2 + J_{P1} + J_{P2} + J_M$

$= (0.0026)Wr^2 + J_{P1} + J_{P2} + J_M$

Friction: $T_F = F_F r$

where:
- w_M = motor speed (rpm)
- v_L = linear load speed (in/min)
- r = pulley radius (in)
- T_L = load torque reflected to motor shaft (lb-in)
- T_F = friction torque (lb-in)
- F_L = load force (lb)
- J_T = total system inertia (lb-in-s^2)
- J_M = motor inertia (lb-in-s^2)
- J_P = pulley inertia(s) (lb-in-s^2)
- W = load weight including belt (lb)
- F_F = frictional force (lb)
- g = gravitational constant (386 in/s^2)