Logic Functions and Symbols

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<tr><th>Truth Table</th><th>Logic Operation</th><th>Boolean Logic Statement</th><th>Schematic Logic Symbol</th></tr>
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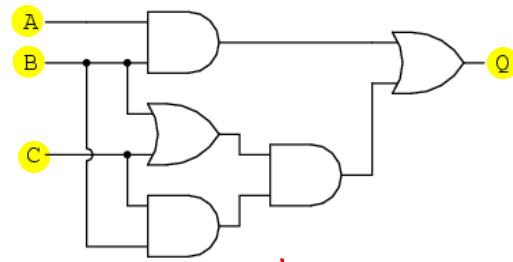
ACIATORE, D.G., HISTAND M.B., INTRODUCTION TO MECHATRONICS AND MEASUREMENT SYSTEMS, 3RD ed., MCGRAW-HILL, NY 2007

6.5 BOOLEAN ALGEBRA

In formulating mathematical expressions for logic circuits, it is important to have knowledge of Boolean algebra, which defines the rules for expressing and simplifying binary logic statements. The basic Boolean laws and identities follow. A bar over a symbol indicates the Boolean operation NOT, which corresponds to inversion of a signal.



COME UP WITH A SIMPLER COMBINATIONAL LOGIC CIRCUIT

Boolean Algebra Laws and Identities

Fundamental Laws

OR	AND	NOT
$A+0=A$	$A\cdot 0=0$	
$A+1=1$	$A\cdot 1=A$	
$A+A=A$	$A\cdot A=A$	$\bar{\bar{A}} = A$ (double inversion)
$A+\bar{A}=1$	$A\cdot \bar{A}=0$	

(6.10)

Commutative Laws

$$A+B = B+A \quad (6.11)$$

$$A\cdot B = B\cdot A \quad (6.12)$$

Associative Laws

$$(A+B)+C = A+(B+C) \quad (6.13)$$

$$(A\cdot B)\cdot C = A\cdot(B\cdot C) \quad (6.14)$$

Distributive Laws

$$A\cdot(B+C) = (A\cdot B)+(A\cdot C) \quad (6.15)$$

$$A+(B\cdot C) = (A+B)\cdot(A+C) \quad (6.16)$$

Other Useful Identities

$$A+(A\cdot B) = A \quad (6.17)$$

$$A\cdot(A+B) = A \quad (6.18)$$

$$A+(\bar{A}\cdot B) = A+B \quad (6.19)$$

$$(A+B)\cdot(A+\bar{B}) = A \quad (6.20)$$

$$(A+B)\cdot(A+C) = A+(B\cdot C) \quad (6.21)$$

$$A+B+(A\cdot\bar{B}) = A+B \quad (6.22)$$

$$(A\cdot B)+(B\cdot C)+(\bar{B}\cdot C) = (A\cdot B)+C \quad (6.23)$$

$$(A\cdot B)+(A\cdot C)+(\bar{B}\cdot C) = (A\cdot B)+(\bar{B}\cdot C) \quad (6.24)$$

De MORGAN'S

$$\overline{A+B+C+\ldots} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\ldots$$

$$\overline{A\cdot B\cdot C\cdot\ldots} = \bar{A}+\bar{B}+\bar{C}+\ldots$$